Abstract

F. V. Gubarev et al. [4] ("On the significance of the vector potential squared") have argued that the minimum value of the volume integral of the vector potential squared may have physical meaning, in defiance of the equivalence of potentials which are connected by the gauge transformations. Earlier, R. I. Khrapko proposed a gauge noninvariant electrodynamics spin tensor [1] ("Spin density of electromagnetic waves"). The standard electrodynamics spin tensor is zero.

Here we point out that the Biot-Savarat formula uniquely results in a preferred, "true" vector potential field which is generated from a given magnetic field. A similar integral formula uniquely permits to find a "true" scalar potential field generated from a given electric field even in the case of a nonpotential electric field.

A conception of differential forms is used. We say that an exterior derivative of a form is the boundary of this form and the integration of a form by the Biot-Savarat formula results in a new form named the generation. Generating from a generation yields zero. The boundary of a boundary is zero. A boundary is closed. A generation is sterile. A conjunction is considered. The conjunction converts closed forms to sterile forms and back. The conjunction permits to construct chains of forms. The conjunction differs from the Hodge star operation: the conjunction does not imply the dualization.

1. The gauge equivalence of differential forms

It is obvious that in a static case we can add a constant ϕ_0 to an electric scalar potential ϕ and we can add a gradient $\partial_i f$ to a magnetic vector potential A_i without changing the corresponding electric E_i and magnetic B_{ij} fields. Indeed,

$$E_i = \partial_i \phi = \partial_i(\phi + \phi_0), \quad B_{ij} = 2\partial_{[i}A_{j]} = 2\partial_{[i}(A_{j]} + \partial_{j]}f).$$

The change

$$\phi \to \phi + \phi_0, \qquad A_i \to A_i + \partial_i f$$

is referred to as the *gauge transformations* of the potentials. Thus, different pairs of potentials ϕ, A_i which are connected by the gauge transformations give the same electromagnetic field E_i, B_{ij}.

All these quantities are differential forms. We name them simply *forms*. All derivatives considered here are external derivatives. We say that an external derivation of a form results in the *boundary* of this form, and we name the form under derivation a *filling* of the boundary. So E_i is the boundary of the form ϕ and B_{ij} is the boundary of the form A_j. ϕ is the filling of the form E_i and A_j is the filling of the form B_{ij}.

The forms ϕ_0 and $\partial_i f$ are referred to as *closed* forms because their external derivatives are equal to zero: $\partial_i\phi_0 = 0$, $\partial_{[i}\partial_{j]}f = 0$. When necessary, we mark closed forms by the symbol bullet $\bullet$: $\underset{\bullet}{\phi}_0$, $\underset{\bullet}{\partial}_j f$.

It is obvious that any boundary is closed. That is, the boundary of a boundary is equal to zero. For example,

$$\partial_{[k}\underset{\bullet}{E}_{i]} = \partial_{[k}\partial_{i]}\phi = 0, \quad \partial_{[k}\underset{\bullet}{B}_{ij]} = 2\partial_{[k}\partial_i A_{j]} = 0.$$

The boundary of a form is determined uniquely, a filling of a form admits an addition of a closed form. Thus, an electric field strength E_i and a magnetic induction B_{ij} do not change when closed

[1]Moscow Aviation Institute, 125871, Moscow, Russia, E-mail: tahir@k804.mainet.msk.su for Khrapko

forms are added to the potentials:

$$\phi \to \phi + \underset{\bullet}{\phi}, \qquad A_i \to A_i + \underset{\bullet}{A}_i.$$

2. True potentials

But, from our point of view, it does not mean that different potentials which are connected by the gauge transformations are completely equivalent to one another. There are preferred, "true" potentials, which correspond to a given electromagnetic field. Early we have stated an assumption that a spin tensor of electromagnetic waves is expressed in terms of such a vector potential [1]. The standard electrodynamics spin tensor is zero.

A true vector potential can be get by the Biot-Savarat formula:

$$A_j(x) = \int \frac{B_{i'j}(x')r^{i'}(x,x')dV'}{4\pi r^3(x,x')}. \tag{1}$$

A true scalar potential can be get by a similar formula:

$$\phi(x) = \int \frac{E_{i'}(x')r^{i'}(x,x')dV'}{4\pi r^3(x,x')}. \tag{2}$$

We use marked indexes. Primes belong to indexes, but not to kernel letters. In the integrals (1), (2) the primes mark a varying point x', but not another coordinate system.

The formula (2) does not meet in literature. It determines the potential ϕ uniquely, in particular, in the case of a nonpotential electric field.

We say that E_i *generates* ϕ by the formula (2) and B_{ij} generates A_i by the formula (1), i.e. we say that ϕ is the *generation from* E_i and A_i is the generation from B_{ij}. Otherwise, we say that E_i is a *source* of ϕ and B_{ij} is a source of A_i. The symbol *dagger* $\dagger$ is used for a brief record of generating, and the symbol *times* $\times$ marks a generation. So, the true potentials are given by the formulas:

$$\underset{\times}{\phi} = \dagger^i E_i, \quad \underset{\times}{A}_j = \dagger^i B_{ij}.$$

Thus, $\dagger$ is an operation which is inverse to an external derivation. Without indexes this looks as follows: $\dagger\partial = \partial\dagger = 1$.

3. Generations are sterile

At this point, a problem arises. What shall we get if a generation will be used as a source of a generation? What shall we get if a generation will be substituted in the integral formula? For example, what is the value of the integral

$$\int \frac{\underset{\times}{A}_{j'}(x')r^{i'}(x,x')dV'}{4\pi r^3(x,x')} \ ?$$

The question is a simple one: generating from a generation yields zero [3]. We say that generations are *sterile*. For example,

$$\int \frac{\underset{\times}{A}_{j'}(x')r^{i'}(x,x')dV'}{4\pi r^3(x,x')} = 0, \qquad \text{briefly: } \dagger^i\dagger^j B_{ij} = 0.$$

It implies that a sterile addition to a source does not change the generation.

Thus,

$$\partial\ (\text{filling}) = \partial\ (\text{filling} + \text{closed form}) = \text{boundary (which is closed)},$$
$$\dagger\ (\text{source}) = \dagger\ (\text{source} + \text{sterile form}) = \text{generation (which is sterile)}$$

Now we can decompose any form into closed and sterile parts. For example:

$$A_k = \underset{\bullet}{A}_k + \underset{\times}{A}_k = \partial_k \dagger^j A_j + \dagger^i 2\partial_{[i}A_{k]},$$

because one can name the expressions

$$\underset{\bullet}{A}_k = \partial_k \dagger^j A_j \quad \text{and} \quad \underset{\times}{A}_k = \dagger^i 2\partial_{[i}A_{k]}$$

closed and sterile components of the form A_k, respectively.

4. The conjugation. Chains of fields

In a metric space there is a relations between contra- and covariant tensors of the same valence (with the same number of indices). For example, the metric tensor g_{ik} associates a tensor X^{ij} with the tensor $X_{mn} = X^{ij} g_{im} g_{jn}$. This process is called the lowering of indices. In this case the same kernel letter for the quantity is used.

In the electromagnetism a slightly different process is used. We call this process the *conjugation*. The conjugation establishes a one-to-one correspondence between forms and contravariant tensor densities. This process uses the metric tensor densities $g^{\wedge}_{ij} = g_{ij}/\sqrt{g}_{\wedge}$ or $g^{ij}_{\wedge} = g^{ij}\sqrt{g}_{\wedge}$. It appears that the electromagnetic fields are conjugated in pairs:

$$E_i = D^j_\wedge g^\wedge_{ij}, \quad D^j_\wedge = E_i g^{ij}_\wedge, \quad B_{ik} = H^{jl}_\wedge g^\wedge_{ij} g_{kl}, \quad H^{jl}_\wedge = B_{ik} g^{ij}_\wedge g^{kl}.$$

As is known, electric induction $D^j_\wedge$, the same as electric charge density $\rho_\wedge$, electric current density $j^i_\wedge$, magnetic strength $H^{ij}_\wedge$ are tensor densities of weight $+1$. To emphasize this circumstance, in serious literature Gothic characters are used for $D^j_\wedge$, $j^i_\wedge$, $H^{ij}_\wedge$. But in the present paper we use the symbol *wedge* $\wedge$ as a sub(super)script to mark a tensor density of weight $+1$ or -1 (see also [3]).

Tensor densities differ from tensors: the law of transformation of density components involves the modulus of Jacobian. For example, an electric induction is transformed according to the formula

$$D^i_\wedge = D^{i'}_{\wedge'} \partial^i_{i'} \mid \Delta' \mid .$$

Here $\partial^i_{i'}$ is the matrix of coordinates transformation: $\partial^i_{i'} = \partial x^i / \partial x^{i'}$. $\Delta' = \mathrm{Det}(\partial^{i'}_i)$ designates the determinant of the inverse matrix.

The kernel letters are usually changed by conjugating of electromagnetic fields. For brevity we designate conjugating by the *star* $\star$:

$$\star E_i = D^i_\wedge, \quad \star D^i_\wedge = E_i, \quad \star B_{ij} = H^{ij}_\wedge, \quad \star H^{ij}_\wedge = B_{ij}.$$

It is remarkable that conjugating transforms sterile fields to closed fields and back [3]. For example,

$$\star\underset{\bullet}{E}_i = \underset{\times}{D}^i_\wedge, \quad \star\underset{\bullet}{D}^i_\wedge = \underset{\times}{E}_i, \quad \star\underset{\bullet}{B}_{ij} = \underset{\times}{H}^{ij}_\wedge, \quad \star\underset{\times}{A}_j = \underset{\bullet}{A}^j_\wedge.$$

By tradition, we have not changed the kernel letter A in the last equality.

So, the true vector potential $\underset{\times}{A}_j$ becomes a closed one, $\underset{\bullet}{A}^j_\wedge$, by conjugating. It implies that $\underset{\bullet}{A}^j_\wedge$ satisfies the Lorentz condition

$$\partial_j \underset{\bullet}{A}^j_\wedge = 0.$$

Conjugating transforms a sterile generation to a closed field, and the new field appears to be ready for new generating. So chains of forms, finite or infinite, arise. We present an example of an infinite chain.

$$\dagger^i \underset{\bullet}{j}{}^i_\wedge = \underset{\times}{H}{}^{ij}_\wedge, \quad \star \underset{\times}{H}{}^{ij}_\wedge = \underset{\bullet}{B}_{ij}, \quad \dagger^i \underset{\bullet}{B}_{ij} = \underset{\times}{A}_j, \quad \star \underset{\times}{A}_j = \underset{\bullet}{A}{}^j_\wedge, \quad \dagger^k \underset{\bullet}{A}{}^j_\wedge = \underset{\times}{\mathcal{H}}{}^{jk}_\wedge, \quad \star \underset{\times}{\mathcal{H}}{}^{jk}_\wedge = \underset{\bullet}{\mathcal{B}}_{jk}, \dots$$

The script characters $\mathcal{H}$ and $\mathcal{B}$ designate hypothetical fields. These fields arise when the chain is constructed. It is another generation.

Conjugating permits recurring derivations. So, a chain can be constructed in the reverse direction by external differentiation. For example:

$$2\partial_{[i}\underset{\times}{A}_{j]} = \underset{\bullet}{B}_{ij}, \quad \star \underset{\bullet}{B}_{ij} = \underset{\times}{H}{}^{ij}_\wedge, \quad \partial_j \underset{\times}{H}{}^{ij}_\wedge = \underset{\bullet}{j}{}^i_\wedge, \quad \star \underset{\bullet}{j}{}^i_\wedge = \underset{\times}{j}_i, \quad 2\partial_{[k}\underset{\times}{j}_{i]} = \underset{\bullet}{\mathrm{B}}_{ki}, \quad \star \underset{\bullet}{\mathrm{B}}_{ki} = \underset{\times}{\mathrm{H}}{}^{ki}_\wedge, \dots$$

The large charactes H and B designate hypothetical fields. These fields arise when the chain is constructed. It is another generation.

Conjugating makes it possible to express the operator ∇^2 in terms of the external derivatives. It appears that

$$\nabla^2 \overset{p}{\omega} = (-1)^p(\star\partial\star\partial - \partial\star\partial\star)\overset{p}{\omega}, \qquad \nabla^2 \overset{p}{\alpha}_\wedge = (-1)^{p+1}(\star\partial\star\partial - \partial\star\partial\star)\overset{p}{\alpha}_\wedge.$$

Here $\overset{p}{\omega}$ and $\overset{p}{\alpha}_\wedge$ designate a form of the degree p and a contravariant density of valence p, respectively. For example,

$$\nabla^2 \underset{\bullet}{A}{}^i_\wedge = -\underset{\bullet}{j}{}^i_\wedge.$$

We denote the integral operator which is inverse to ∇^2 by *double dagger* $\ddagger$. As is known,

$$\ddagger = -\int \frac{dV'}{4\pi r(x, x')}.$$

The requirement $\nabla^2 \ddagger = 1$ yields

$$\ddagger \overset{p}{\omega} = (-1)^{p+1}(\star\dagger\star\dagger - \dagger\star\dagger\star)\overset{p}{\omega}, \qquad \ddagger \overset{p}{\alpha}_\wedge = (-1)^p(\star\dagger\star\dagger - \dagger\star\dagger\star)\overset{p}{\alpha}_\wedge.$$

For example,

$$\ddagger \underset{\bullet}{j}{}^i_\wedge = -\underset{\bullet}{A}{}^i_\wedge, \qquad \ddagger \underset{\bullet}{B}_{ij} = -\underset{\bullet}{\mathcal{B}}_{ij}.$$

5. Vector potential squared

The article [4] is an occasion for this paper writing. The authors of the article [4] “argue that the minimum value of the volume integral of A^2 may have physical meaning”. In other words, the potential which minimizes the volume integral is a preferred potential. The authors have designated such a potential A_{min}.

We have named such a potential “true” potential: $\underset{\times}{A}_j = \dagger^i 2\partial_{[i}A_{k]}$. Therefore, the mentioned volume integral should be evaluated by the formula

$$< \underset{\times}{A}_j \cdot \star \underset{\times}{A}_j >=< \underset{\times}{A}_j \cdot \underset{\bullet}{A}{}^j_\wedge >= \int \underset{\times}{A}_j \underset{\bullet}{A}{}^j_\wedge dV^\wedge$$

($dV^\wedge$ is a density of weight -1).

However, the authors use another formula:

$$< A^2_{\min} >= \int\int \frac{\mathbf{B}(x')\cdot\mathbf{B}(x)dVdV'}{4\pi r(x,x')}.$$

This formula can be obtained by transforming the expression $< \underset{\times}{A}_j \cdot \underset{\bullet}{A}{}^j_\wedge >$:

$$< \underset{\times}{A}_j \cdot \underset{\bullet}{A}{}^j_\wedge >=< \underset{\times}{A}_j \cdot \partial_k \underset{\times}{\mathcal{H}}{}^{jk}_\wedge >= -2 < \partial_{[k} \underset{\times}{A}_{j]} \cdot \underset{\times}{\mathcal{H}}{}^{jk}_\wedge >=< \underset{\bullet}{B}_{jk} \cdot \star \underset{\bullet}{\mathcal{B}}_{jk} >= - < \underset{\bullet}{B}_{jk} \cdot \star\ddagger \underset{\bullet}{B}_{jk} > .$$

It is sad that the authors of [4] call *rest mass* the mass. Actually, mass is the equivalent of inertia of a body and varies with speed of the body [5, 6].

6. The standard electrodynamics spin tensor is zero

The energy-momentum tensor $T^{\alpha\gamma}$ and the spin tensor $\Upsilon^{\alpha\gamma\beta}$ (upsilon) are defined by the following equalities:

$$dP^\alpha = T^{\alpha\gamma}dV_\gamma, \qquad dS^{\alpha\gamma} = \Upsilon^{\alpha\gamma\beta}dV_\beta \qquad \alpha,\gamma,\ldots = 0,1,2,3.$$

Here infinitesimal 4-momentum dP^α and 4-spin $dS^{\alpha\gamma}$ are observable quantities and dV_γ is an 3-element. So true definitions of the energy-momentum and spin tensors do not admit any arbitrariness.

The electrodynamic energy-momentum tensor is the Minkowski tensor.

$$T^{\alpha\gamma} = -F^{\alpha\nu}F^\gamma{}_\nu + g^{\alpha\gamma}F_{\nu\mu}F^{\nu\mu}/4.$$

Only this tensor satisfies experiments. Only this tensor localizes energy-momentum. The source of the Minkowski tensor is

$$\partial_\gamma T^{\alpha\gamma} = -F^\alpha_\gamma j^\gamma.$$

The Minkowski tensor is the true electrodynamic energy-momentum tensor. But a true spin tensor in the electrodynamics is unknown,

$$\Upsilon^{\alpha\gamma\beta} = ?$$

In the electrodynamics the variational principle results in a pair of the canonical tensors: the canonical energy-momentum tensor $\underset{c}{T}{}^{\alpha\gamma}$ and the canonical spin tensor $\underset{c}{\Upsilon}{}^{\alpha\gamma\beta}$ (upsilon):

$$\underset{c}{T}{}^{\alpha\gamma} = -\partial^\alpha A_\mu \cdot F^{\gamma\mu} + g^{\alpha\gamma}F_{\mu\nu}F^{\mu\nu}/4, \qquad \underset{c}{\Upsilon}{}^{\alpha\gamma\beta} = -2A^{[\alpha}F^{\gamma]\beta}.$$

These tensors contradict experience. It is obvious in view of a asymmetry of the energy-momentum tensor, and it was checked on directly [2].

An attempt is known to turn the canonical energy-momentum tensor to the Minkowski tensor by subtraction the Rosenfeld's pair of tensors,

$$(\underset{R}{T}{}^{\alpha\gamma}, \quad \underset{R}{\Upsilon}{}^{\alpha\gamma\beta}) = (\partial_\beta \underset{c}{\Upsilon}{}^{\{\alpha\gamma\beta\}}/2, \quad \underset{c}{\Upsilon}{}^{\alpha\gamma\beta}), \qquad \Upsilon^{\{\alpha\gamma\beta\}} = \Upsilon^{\alpha\gamma\beta} - \Upsilon^{\gamma\beta\alpha} + \Upsilon^{\beta\alpha\gamma},$$

from the canonical pair of tensors.

The Rosenfeld's pair is closed in the sense:

$$\partial_\gamma \underset{R}{T}{}^{\alpha\gamma} = 0, \qquad \partial_\beta \underset{R}{\Upsilon}{}^{\alpha\gamma\beta} = 2\underset{R}{T}{}^{[\alpha\gamma]}.$$

So, the Rosenfeld's pair is closed relative both momentum and spin. This implies that external sources of the Rosenfeld's pair are zero.

Subtracting the Rosenfeld's pair yields

$$\underset{c}{T}^{\alpha\gamma} - \underset{R}{T}^{\alpha\gamma} = T^{\alpha\gamma} - A^{\alpha} j^{\gamma}, \qquad \underset{c}{\Upsilon}^{\alpha\gamma\beta} - \underset{R}{\Upsilon}^{\alpha\gamma\beta} = 0.$$

So, the subtraction eliminates the spin tensor and, in the case of $j^{\gamma} = 0$, yields the Minkowski energy-momentum tensor.

The elimination of the electrodynamic spin tensor provokes a strange opinion that a circularly polarized plane wave with infinite extent can have no angular momentum [7, 8], that only a quasiplane wave of finite transverse extent carries an angular momentum whose direction is along the direction of propagation. This angular momentum is provided by an outer region of the wave within which the amplitudes of the electric E and magnetic B fields are decreasing. These fields have components parallel to wave vector there, and the energy flow has components perpendicular to the wave vector. "This angular momentum is the spin of the wave" [9]. Within an inner region the E and B fields are perpendicular to the wave vector, and the energy-momentum flow is parallel to the wave vector [10]. There is no angular momentum in the inner region [9].

But let us suppose now that a circularly polarized beam is absorbed by a round flat target which is divided concentrically into outer and inner parts. According to the previous reasoning, the inner part of the target will not perceive a torque. Nevertheless R. Feynman [11] clearly showed how a circularly polarized plane wave transfers a torque to an absorbing medium. What is true? And if R. Feynman is right, how one can express the torque in terms of pondermotive forces?

From our point of view, classical electrodynamics is not complete. The task is to discover the nonzero spin tensor of electromagnetic field.

7. Spin tensor of electromagnetic waves

For getting the Minkowski tensor from the canonical tensor in the general case of $j^{\gamma} \neq 0$ we have to subtract

$$\tilde{T}^{\alpha\gamma} = \underset{R}{T}^{\alpha\gamma} - A^{\alpha} j^{\gamma}$$

from $\underset{c}{T}^{\alpha\gamma}$:

$$T^{\alpha\gamma} = \underset{c}{T}^{\alpha\gamma} - \tilde{T}^{\alpha\gamma}.$$

What tensor $\tilde{\Upsilon}^{\alpha\gamma\beta}$ must we then subtract from $\underset{c}{\Upsilon}^{\alpha\gamma\beta}$ for getting the true spin tensor?

We suggested that

$$\tilde{\Upsilon}^{\alpha\gamma\beta} = \underset{c}{\Upsilon}^{\alpha\gamma\beta} - 2A^{[\alpha}\partial^{|\beta|}A^{\gamma]}$$

because such a $\tilde{\Upsilon}^{\alpha\gamma\beta}$ is closed relative to spin:

$$\partial_{\beta}\tilde{\Upsilon}^{\alpha\gamma\beta} = 2\tilde{T}^{[\alpha\gamma]}.$$

This way we obtain an *electric* spin tensor:

$$\underset{e}{\Upsilon}^{\alpha\gamma\beta} = \underset{c}{\Upsilon}^{\alpha\gamma\beta} - \tilde{\Upsilon}^{\alpha\gamma\beta} = 2A^{[\alpha}\partial^{|\beta|}A^{\gamma]}.$$

The electromagnetic covariant tensor field $F_{\mu\nu}$ is closed:

$$\partial_{[\alpha}F_{\mu\nu]} = 0$$

. But, for waves the conjugate tensor is closed too (we omit wedge in Sec. 6, 7, 8):

$$\partial_\nu(\star F_{\mu\nu}) = \partial_\nu F^{\mu\nu} = j^\mu = 0.$$

Therefore $F^{\mu\nu}$ has a filling, $\Pi^{\mu\nu\sigma}$,

$$\partial_\sigma \Pi^{\mu\nu\sigma} = F^{\mu\nu}.$$

We call $\Pi^{\mu\nu\sigma}$ an *electric 3-vector potential.*

The "Lorentz condition", $\partial\star(\Pi^{\mu\nu\sigma}) = 0$, singles an electric vector potential out from the collection of the gauge equivalent potentials. We call an electric vector potential the true potential if $\partial_{[\lambda}\Pi_{\mu\nu\sigma]} = 0$.

$\Pi^\alpha = \epsilon^{\alpha\mu\nu\sigma}\Pi_{\mu\nu\sigma}$ and A^α are equal in rights. So the spin tensor must be symmetric relative to the magnetic and the electric potentials. Therefore we suggested that the spin tensor of electromagnetic waves is the sum:

$$\Upsilon^{\alpha\gamma\beta} = \underset{e}{\Upsilon^{\alpha\gamma\beta}} + \underset{m}{\Upsilon^{\alpha\gamma\beta}} = A^{[\alpha}\partial^{|\beta|}A^{\gamma]} + \Pi^{[\alpha}\partial^{|\beta|}\Pi^{\gamma]}. \tag{3}$$

8. Circularly polarized standing wave

Let us consider a circularly polarized plane wave which falls upon a superconducting x, y-plain, and reflects from it, and so a standing wave forms. The flux density of energy (or volumetric momentum density) is equal to zero in the wave, $T^{tz} = \mathbf{E} \times \mathbf{B} = 0$. But the volumetric densities of electrical and magnetic energy vary with z in anti-phase. So the total energy density is constant. The momentum flux density, that is the pressure, is constant too:

$$E^2/2 = 1 - \cos 2kz, \quad B^2/2 = 1 + \cos 2kz, \quad T^{tt} = T^{zz} = (E^2 + B^2)/2 = 2.$$

It is interesting to calculate an output of the expression (3) in the situation. The spin flux density must be zero, $\Upsilon^{xyz} = 0$, and it is expected that the volumetric spin density consists of electrical and magnetic parts which are shifted relative to one another. This result is obtained below.

A circularly polarized plane wave which propagates along z-direction involves the vectors $\mathbf{B}$, $\mathbf{E}$, $\mathbf{A}$, Π which lay in xy-plane, and we shall represent them by complex numbers instead of real parts of complex vectors.

$$\mathbf{B} = \{B^x,\ B^y\} \rightarrow\ B = B^x + iB^y.$$

Then the product of a complex conjugate number $\overline{E}$ and other number B is expressed in terms of scalar and vector products of the corresponding vectors. For example:

$$\overline{E} \cdot B = (\mathbf{E} \cdot \mathbf{B}) + i(\mathbf{E} \times \mathbf{B})^z.$$

Since all this vectors do not vary with x and y, then

$$\mathrm{curl}\mathbf{B} = \{-\partial_z B^y,\ \partial_z B^x\} \rightarrow i\partial_z B, \quad \mathrm{curl}^{-1} \rightarrow -i\int dz.$$

The angular velocity of all the vectors is ω and the wave number along z-axis is $k = \omega$. Therefore

$$\mathbf{B} \rightarrow B_{01}e^{i\omega(t-z)} \text{ or, for a reflected wave, } B_{02}e^{i\omega(t+z)},$$

$$\partial_t \rightarrow i\omega, \quad \partial_z \rightarrow \mp i\omega, \quad \mathrm{curl} \rightarrow \pm\omega, \quad \mathrm{curl}^{-1} \rightarrow \pm 1/\omega.$$

If $z = 0$ at the superconducting x, y-plain, then the falling and reflected waves are recorded as

$$B_1 = e^{i\omega(t-z)}, \quad E_1 = -ie^{i\omega(t-z)}, \quad B_2 = e^{i\omega(t+z)}, \quad E_2 = ie^{i\omega(t+z)}.$$

The complex amplitudes are equal here: $B_{01} = B_{02} = 1,\ E_{01} = -i,\ E_{02} = i$.

Since $\mathbf{A} = \mathrm{curl}^{-1}\mathbf{B},\ \Pi = \mathrm{curl}^{-1}\mathbf{E}$, the other complex amplitudes are received by a simple calculation (time derivative is designated by a point):

$$A_{01} = 1/\omega,\ \dot{A}_{01} = i,\ \Pi_{01} = -i/\omega,\ \dot{\Pi}_{01} = 1,\ A_{02} = -1/\omega,\ \dot{A}_{02} = -i,\ \Pi_{02} = -i/\omega,\ \dot{\Pi}_{02} = 1.$$

Now we calculate the electric and magnetic parts of the volumetric spin density.

$$\begin{aligned} \underset{e}{\Upsilon}^{xyt} &= (\mathbf{A} \times \dot{\mathbf{A}})/2 = \Im(\overline{(A_1 + A_2)} \cdot (\dot{A}_1 + \dot{A}_2))/2 \\ &= \Im((e^{-i\omega(t-z)} - e^{-i\omega(t+z)})i(e^{i\omega(t-z)} - e^{i\omega(t+z)}))/2\omega = (1 - \cos 2\omega z)/\omega, \end{aligned}$$

$$\underset{m}{\Upsilon}^{xyt} = (\Pi \times \dot{\Pi})/2 = \Im(\overline{(\Pi_1 + \Pi_2)} \cdot (\dot{\Pi}_1 + \dot{\Pi}_2))/2 = (1 + \cos 2\omega z)/\omega,$$

$$\Upsilon^{xyt} = \underset{e}{\Upsilon}^{xyt} + \underset{m}{\Upsilon}^{xyt} = 2/\omega.$$

So, the terms which oscillate along z-axis cancel out. It is easy to calculate that spin flux is equal to zero (the prime denote the derivative with respect to z):

$$\underset{e}{\Upsilon}^{xyz} = -(\mathbf{A} \times \mathbf{A}')/2 = 0, \quad \underset{m}{\Upsilon}^{xyz} = -(\Pi \times \Pi')/2 = 0.$$

Note

This paper matter is contained in the following papers which were submitted to the following journals (all the journals rejected or ignored all the papers):

"Electromagnetism in terms of sources and generations of fields" *Physics - Uspekhi* (13 June, 1995).

"Electromagnetism: sources, generations, boundaries", "Spin tensor of electromagnetic fields" *J. Experimental & Theor. Phys. Lett.* (14 May, 1998).

"Spin tensor of electromagnetic fields" *J. Experimental & Theor. Phys.* (27 Jan. 1999), *Theor. Math. Phys.* (29 Apr. 1999), *Rus. Phys. J.* (18 May, 1999).

"Are spin and orbital momentum the same quantity?" *J. Experimental & Theor. Phys.*, *Physics - Uspekhi* (25 Feb. 1999), *Rus. Phys. J.* (15 Oct. 1999).

"Does plane wave not carry a spin?" *AJP* (7 Oct. 1999)

"Energy-momentum and spin tensors problems in the electromagnetism" *J. Experimental & Theor. Phys.* (13 Apr. 2000), *Physics - Uspekhi* (12 Jan. 2000), *Rus. Phys. J.* (1 March. 2000), *Theor. Math. Phys.* (17 Feb. 2000).

"Angular momentum distribution of the rotating dipole field" *J. Experimental & Theor. Phys.*, *Rus. Phys. J.* (25 May, 2000), *Theor. Math. Phys.* (29 May, 2000). *Physics - Uspekhi* (31 May, 2000).

"Electromagnetism in terms of boundaries and generations of differential forms" *Physics - Uspekhi* (4 Oct. 2000).

"Tubes of force and bisurfaces in the electromagnetism" *Physics - Uspekhi* (28 March, 2001), *Rus. Phys. J.* (26 Apr. 2001).

The subject matter of this paper had been partially published [1–3, 12].

References

1. Khrapko R. I., “Spin density of electromagnetic waves”, http://www.mai.ru/projects/mai_works/index.htm

2. Khrapko R. I., “True energy-momentum tensors are unique. Electrodynamics spin tensor is not zero”, physics/0102084.

3. Khrapko R. I., “Tubes of force and bisurfaces in electromagnetism”, http://www.mai.ru/projects/mai_works/index.htm

4. Gubarev F. V., Stodolsky L., Zakharov V. I., “On the Significance of the Vector Potential Squared”, Phys. Rev. Lett., **86**, 2220 (2001)

5. Khrapko R. I., “What is mass?”, physics/0103051, Physics - Uspekhi **43** (12) 1267 (2000)

6. Khrapko R. I., “Rest mass or inertial mass?”, physics/0103008.

7. Heitler W., The Quantum Theory of Radiation (Clarendon, Oxford, 1954), p. 401.

8. Simmonds J. W., Guttmann M. J., States, Waves and Photons (Addison- Wesley, Reading, MA, 1970).

9. Ohanian H. C., “What is spin?” Amer. J. Phys. **54**, 500, (1986).

10. Jackson J. D., Classical Electrodynamics (John Wiley, New York, 1962), p. 201.

11. Feynman R. P., *et al.* The Feynman Lectures On Physics (Addison-Wesley, London, 1965), v. 3, p. 17-10.

12. Khrapko R. I., “True energy-momentum and spin tensors are unique.” *In* Abstracts of 10-th Russian Gravitational Conference (Moscow, 1999), p. 47.